ÖAW ÖSTERREICHISCHE AKADEMIE DER WISSENSCHAFTEN

universität wien

Workshop „*Inverse Problems in the Alps II*“

Obergurgl, AT, 21 – 23 March 2018

Scientific Program

Wednesday, 21 March 2018

09:00 – 09:30 ***Laser beam imaging from the speckle pattern of the off-axis scattered in Tensity****, Liliana Borcea, University of Michigan*

We study the inverse problem of localization (imaging) of a laser beam from measurements of the intensity of light scattered off-axis by a Poisson cloud of small particles. Starting from the wave equation, we analyze the microscopic coherence of the scattered intensity and show that it is possible to determine the laser beam from the speckle pattern captured by a group of CCD cameras. Two groups of cameras are sufficient when the particles are either small or large with respect to the wavelength. For general particle sizes the accuracy of the laser localization with two groups of cameras is subject to knowing the scattering properties of the cloud. However, three or more groups of cameras allow accurate localization that is robust to uncertainty of the type, size, shape and concentration of the particles in the cloud. We introduce a novel laser beam localization algorithm and give some numerical illustrations in a regime relevant to the application of imaging high energy lasers in a maritime atmosphere.

09:30 – 10:00 ***All-at-once versus reduced iterative methods for time dependent inverse Problems***
Barbara Kaltenbacher, Alpen-Adria-Universität Klagenfurt

A large number of inverse problems in applications ranging from engineering via economics to systems biology can be formulated as a state space system with a finite or infinite dimensional parameter that is supposed to be identified from additional continuous or discrete observations. The aim of this talk is to formulate inverse problems of this kind in a reduced and an all-at-once fashion and compare some iterative regularization methods in these two different settings as they have already been shown to exhibit crucial differences for stationary inverse problems. In particular, we will consider Landweber iteration and the iteratively regularized Gauss-Newton method, as well as the Landweber-Kaczmarz iteration, as the latter obviously lends itself to a subdivision of the problem to a union of time intervals.

10:00 – 10:30 ***Imaging small polarizable scatterers with polarization data***
Fernando Guevara Vasquez, University of Utah

We consider the problem of imaging small scatterers by using polarization data (e.g. Stokes parameters) measured at an array of receivers. This data is obtained by illuminating the scatterers with a point source for which we only assume its polarization can be controlled. We show that the polarization data can be preprocessed so that it can be used to image the scatterers using an electromagnetic version of Kirchhoff migration. The images that are produced are matrix valued and are asymptotically indistinguishable from the images obtained from measuring the phase and amplitude of all three components of the electric field at the array. The Kirchhoff images can be used to estimate a projection of the polarization matrix of the scatterers in an appropriate basis.

10:30 – 11:00 *** *Coffee Break* ***

11:00 – 11:30 ***Acoustic Imaging and Multiple Removal via Model Order Reduction***
Alex Mamonov, University of Houston

We introduce a novel framework for acoustic imaging and removal of multiples from full waveform data based on model order reduction. The reduced order model (ROM) is an orthogonal projection of the wave equation propagator (Green's function) on the subspace of discretely sampled time domain wavefield snapshots. Even though neither the propagator nor the wavefields are known in the bulk, the projection can be computed just from the knowledge of the waveform data using the block Cholesky factorization. Once the ROM is found, its use is twofold.

First, the projected propagator can be backprojected to obtain an image of reflectors. ROM computation implicitly orthogonalizes the wavefield snapshots. This highly nonlinear procedure differentiates our approach from the conventional linear migration methods (Kirchhoff, RTM). It allows to resolve the reflectors independently of the knowledge of the kinematics and to untangle the nonlinear interactions between the reflectors. As a consequence, the resulting images are almost completely free from the multiple reflection artifacts.

Second, the ROM computed from the original full waveform data can be used to generate the Born data set, i.e. the data that the measurements would produce if the propagation of waves in the unknown medium obeyed Born approximation instead of the wave equation. Obviously, such data only contains primary reflections and the multiples are removed with their energy mapped back to the primaries. Consecutively, existing linear imaging and inversion techniques can be applied to Born data to obtain reconstructions in a direct, non-iterative manner.

11:30 – 12:00 ***Coherent Acousto-Optic Imaging***
John Schotland, University of Michigan

*** *Break, Dinner 18:00 –19:15* ***

19:15 – 19:45 ***Improving FDK Reconstruction by Data-Dependent Filtering***
Rien Lagerwerf, CWI, Netherlands

The FDK method is the most common reconstruction method in 3D circular cone beam computed tomography.

A limitation of the FDK method is that its reconstruction accuracy degrades if the number of projection angles is small or if the data has a high noise level. In this case, iterative methods are preferred as they can balance between fitting the available data and regularizing the solution. However for large datasets their substantial computational cost compared to the FDK method is still a major concern if high throughput is required. In this paper we propose the Algebraic Filter FDK method (AF-FDK), which aims to combine the computational efficiency of FDK with the accuracy of iterative methods. The AF-FDK method computes an algebraic filter specific to the measured projection data, such that the resulting FDK reconstruction fits well with the data.

We show for simulated and experimental data that the AF-FDK method results in more accurate reconstructions compared to the FDK method with standard filters when faced with limited or noisy data, while being competitive with the accuracy of the iterative SIRT method. Additionally we show that the algebraic filter computed for a specific dataset can be used to reconstruct datasets of other objects acquired under similar conditions without losing reconstruction quality. This means that once an algebraic filter has been computed, subsequent FDK reconstructions achieve a favorable combination of the computational efficiency of standard FDK with the reconstruction accuracy of iterative methods.

19:45 – 20:15 ***The reconstruction of a source and a potential from boundary measurements****, Amin Boumenir, University of West Georgia*

We are concerned with the reconstruction of both the heat coefficient and the source appearing in a parabolic equation from observations of the solution on the boundary. We propose two methods. The first uses a single point on the boundary and a final time overdetermination to recover the source and is based on properties of the principal Dirichlet eigenfunction. The second method uses only the observations over a small region on the boundary and is based on Holmgren's theorem.

20:15 – 20:45 ***Signal Analysis and Reconstruction Algorithms in 2D Computed Tomography****, Adel Faridani, Oregon State University*

Computed tomography produces images from the interior of opaque objects. A variety of data acquisition geometries and reconstruction algorithms are available for practical use. In this talk we will use Signal Analysis in the sense of applying the Shannon Sampling Theorem and its generalizations in order to identify potentially efficient sampling schemes that would allow reconstruction with a minimal amount of data. For various data acquisition schemes we will then identify and analyze suitable numerical reconstruction algorithms that achieve the desired performance.

Thursday, 22 March 2018

09:00 – 10:00 ***Nonlinear responses from the interaction of two progressing waves at an interface***
Maarten de Hoop, Rice University

For scalar semilinear wave equations, we analyze the interaction of two (distorted) plane waves at an interface between media of different nonlinear properties. We show that new waves are generated from the nonlinear interactions. Furthermore, we show that the incident waves and the nonlinear responses determine the location of the interface and some information of the nonlinear properties of the media. In particular, for the case of a jump discontinuity at the interface, we can determine the magnitude of the jump in the nonlinear parameter. We will briefly indicate the generalization to nonlinear elastodynamics.

Joint research with Gunther Uhlmann and Yiran Wang

10:00 – 10:30 ***Stability in Inverse Problems via Unique Continuation Properties***
Sergio Vesella, Universitá Degli Studi Firenze

The issue of stability estimates is of fundamental importance in constructive methods for inverse problems and the quantitative estimates of unique continuation ***play an important role*** in such an issue. In this talk I would like to present some recent results of quantitative estimates of unique continuation concerning the transmission problems for elliptic and parabolic equations

10:30 – 11:00 *Coffee Break*

11:00 – 11:30 ***On an elastic model arising from volcanology: an analysis of the direct and inverse problem***
Andrea Aspri, RICAM Linz

In this talk I will briefly describe a linear elastic model for the detection of magma chambers. This model describes the surface deformation effects generated by a magma chamber which exerts a uniform hydrostatic pressure on the surroding crust. The modeling assumptions translate mathematically into a Neumann boundary value problem for the classical Lamé system in a half-space with an embedded pressurized cavity. I will briefly explain how to get the well-posedness of this problem in weighted Sobolev spaces. Then I will give some details on the analysis of the inverse problem of determing the pressurized cavity from partial measurements of the displacement field on the boundary of the half-space. Specifically I will sketch how to get stability estimates for the cavity. This is a joint work with Elena Beretta and Edi Rosset.

11:30 – 12:00 ***A transmission problem on a polygonal partition***
Elisa Francini, University of Florence

We consider a transmission problem on polygonal partition: we deal with regularity of the solution and we calculate its shape derivative with respect to movements of the partition. We also use this derivative to obtain a stability result for the inverse problem of recovering the partition from knowledge of boundary data.

12:00 – 12:30 ***An inverse problem for a semilinear parabolic equation arising in cardiac electrophysiology***
Cristina Cerutti, Politecnico di Milano

In this talk we deal with a model arising in the study of electrical activity in the heart involving a semilinear equation (the {\em monodomain model}).

The goal is the detection of an inhomogeneity (where the coefficients of the equation are altered) located inside a given domain starting from observations of the potential on the boundary of the domain.

Such a problem is related to the detection (at early stages of their development) of myocardial ischemic regions, characterized by severely reduced blood perfusion and consequent lack of electric conductivity, from noninvasive (or minimally invasive) measurements of electrical activity of the heart.

In joint work with E. Beretta, C. Cavaterra, A. Manzoni and L. Ratti we develop theoretical analysis and numerical reconstruction techniques for the solution of an inverse boundary value problem: following an approach similar to the one introduced by Capdeboscq and Vogelius (2003) in the case of the linear conductivity equation, we provide an asymptotic formula for electric potential perturbations caused by internal conductivity inhomogeneities of low volume fraction. Then we will show numerical reconstructions based on the topological gradient of a suitable cost functional.

Break, Dinner 18:00 –20:00

Friday, 23 March 2018

09:00 – 09:30 ***On the seismic inverse problem: uniqueness, stability and reconstruction***
Elena Beretta, Politecnico di Milano

Reflection seismology is an imaging technique used to determine properties of the earth's subsurface from reflected seismic waves generated by controlled sources like, for example, vibroesis. Models of seismic wave propagation lead to pdes or systems of pdes where the goal is to identify the coefficients containing information on the mechanical properties of rock (density, stiffness) from overdetermined data. Thus, reflection seismology imaging translates mathematically in terms of an inverse boundary value problem for pdes. In my talk I will focus the attention on the case of time harmonic elastic waves. Seismic data from land acquisition can be represented by the Neumann to Dirichlet map. So, we end up with an inverse problem for the time harmonic elastic wave equation where, assuming small-amplitude deformations and isotropy, one seeks to identify the Lam ́e parameters and the density from knowledge of the Neumann to Dirichlet or equivalently of the Dirichlet to Neumann map.

First, I will overview the main known results concerning uniqueness. In the second part of the talk I will concentrate on the issue of continuous dependence, crucial for effective reconstruction. I will illustrate the results contained in [1] where Lipschitz continuous dependence estimates have been derived in the case of piecewise constant elasticity isotropic tensor and density on a known polyhedral partition of the background medium and I will give some highlights on the main tools of the proof.

In the last part of the talk, I will describe an iterative reconstruction algorithm based on the minimization of a suitable misfit functional and I will show some numerical results that assess the effectiveness of the algorithm in identifying the unknown parameters, [2].

References

[1] E. Beretta, M. V. de Hoop, E. Francini, S. Vessella, J Zhai " Conditional Lipschitz stability of an inverse boundary value problem for the time harmonic elastic waves.", Vol 33, Number 3, (2017) Inverse Problems

[2] J. Shi, E. Beretta, M. V. de Hoop, E. Francini, S. Vessella, "Multi-parameter iterative reconstruction with the multi-frequency Neumann-to-Dirichlet map as the data", submitted to Geophys. J. Int. (2018)

09:30 – 10:00 ***Ill-posedness concepts in Hilbert spaces and first steps for handling over smoothing penalties***
Bernd Hofmann, TU Chemnitz

In the first part, we consider different concepts of well-posedness and ill-posedness and their relations for solving non-linear and linear operator equations in Hilbert spaces. First, the concepts of Hadamard and Nashed are recalled which are appropriate for linear operator equations. For non-linear operator equations, stable respective unstable solvability is considered, and the properties of local well-posedness and ill-posedness are investigated. Those two concepts consider stability in image space and solution space, respectively, and both seem to be appropriate concepts for non-linear operators which are not onto and/or not, locally or globally, injective.

Several example situations for non-linear problems are considered, including the prominent autoconvolution problems and other quadratic equations in Hilbert spaces.

In the second part, we study Tikhonov regularization for ill-posed non-linear operator equations in Hilbert scales. Our focus is on the interplay between the smoothness-promoting properties of the penalty and the smoothness inherent in the solution. The objective is to study the situation when the unknown solution fails to have a finite penalty value, hence when the penalty is oversmoothing. By now this case was only studied for linear operator equations in Hilbert scales. We extend those results to certain classes of non-linear problems. The main result asserts that, under appropriate assumptions, order optimal reconstruction is still possible.

This talk presents joint work with Peter Mathé (Berlin) and Robert Plato (Siegen). Regularization with oversmoothing penalties is also a subtopic of the upcoming Austrian/German joint research project "Novel Error Measures and Source Conditions of Regularization Methods for Inverse Problems (SCIP)" with Otmar Scherzer (Vienna) based on the D-A-CH Lead-Agency Agreement. This research is supported by the Deutsche Forschungsgemeinschaft (DFG) under grant HO 1454/12-1.

10:00 – 10:30 ***Topological derivatives for domain functionals with an application to tomography***

Esther Klann, TU Berlin

We study the topological sensitivity of the piecewise constant Mumford–Shah type functional for linear ill-posed problems. We consider a linear operator $K : X \to Y$ and noisy data g^δ approximating $g = K f$ where f is the function we are interested in. We assume $f : D \to R$, $D \subset R^2$ and

$$f = \sum_{i=1}^{m} c_i \, \chi\Omega_i \quad \text{with} \quad c_i \in \mathbb{R}, \qquad \Omega_i \subset \mathbb{R}^2 \quad \text{and} \quad \chi D = \sum_{i=1}^{m} \chi\Omega_i$$

i.e., f is a piecewise constant function and the sets Ω_i are a partition of the domain of definition D. We study the topological sensitivity of the Mumford–Shah-type functional

$$J(\underset{C}{\rightarrow},\underset{\Omega}{\rightarrow}) := \left\|Kf - g^\delta\right\|_{L2}^2 + \alpha \sum_{i=1}^{m} |\partial\Omega_i|$$

i.e., its reaction to a change in topology such as inserting or removing a set Ωj . The topological derivative indicates if such a change in the topology will decrease the value of the Mumford-Shah-type functional, thus it can be used to find a minimizer of the functional and a solution to the reconstruction problem.

We use the topological derivative in an application from tomographic imaging (with the Radon transform as operator) to find inclusions in an object.

References

[1] R. Ramlau and W. Ring. A Mumford-Shah approach for contour tomography. Journal of Computational Physics, Volume 221, Issue 2, 10 February 2007, Pages 539-557.

[2] M. Hintermüller and A. Laurain. Multiphase Image Segmentation and Modulation Recovery based on Shape and Topological Sensitivity. Journal of Mathematical Imaging and Vision, September 2009, Volume 35, Issue 1, pp 1-22

10:30 – 11:00 *** *Coffee Break* ***

11:00 – 11:30 ***A variational reconstruction method for dynamical X-ray tomography***
Tapio Helin, University of Helsinki

We study a variational approach that incorporates physical motion models and regularization of motion vectors to achieve improved reconstruction results for dynamical X-ray tomography in two dimensions. We focus on realistic measurement protocols for practical applications, i.e. we do not assume to have a full Radon transform in each time step, but only projections in few angular directions, this restriction enforces a space-time reconstruction.

In particular we utilize the methodology of optical flow, which is one of the most common methods to estimate motion between two images taken at different time instances. The resulting minimization problem is then solved in an alternating way. Results are presented for simulated and real measurement data.

This work is a joint project with: M. Burger, H. Dirks, L. Frerking, A. Hauptmann and S. Siltanen.

11:30 – 12:00 ***Using Landwebers method to quantify source conditions - a numerical study***
Daniel Gerth, TU Chemnitz, Germany

The theory for the regularization of bounded, linear, and compact operators between Hilbert spaces is well established. Unfortunately, the results can not always be carried over in practice as the required parameters are often not available. Namely, one essentially needs information on the magnitude of the noise in the data and a parameter describing the smoothness of the exact solution to the noise-free problem. This in particular renders a-priori choices for the regularization parameter often useless in practice, but even a-posteriori methods like the discrepancy principle may not be justified anymore. In this talk, we focus on the smoothness parameter μ from the classical source condition $x \in R(A^* A)^{\mu}$. We demonstrate that it is possible to estimate μ from the norms of residual and gradient obtained throughout a Landweber iteration. We show results of the algorithm for various numerical examples ranging from a purely academical test setting to a real world measurement situation. The theory behind the algorithm is based on a Lojasiwiecz-type inequality which we show is implied by the source condition.

12:00 – 13:00 *** *Closing Lunch* ***